

# Mathematics (Erasmus+ Project)

## GeoGebra Workshop

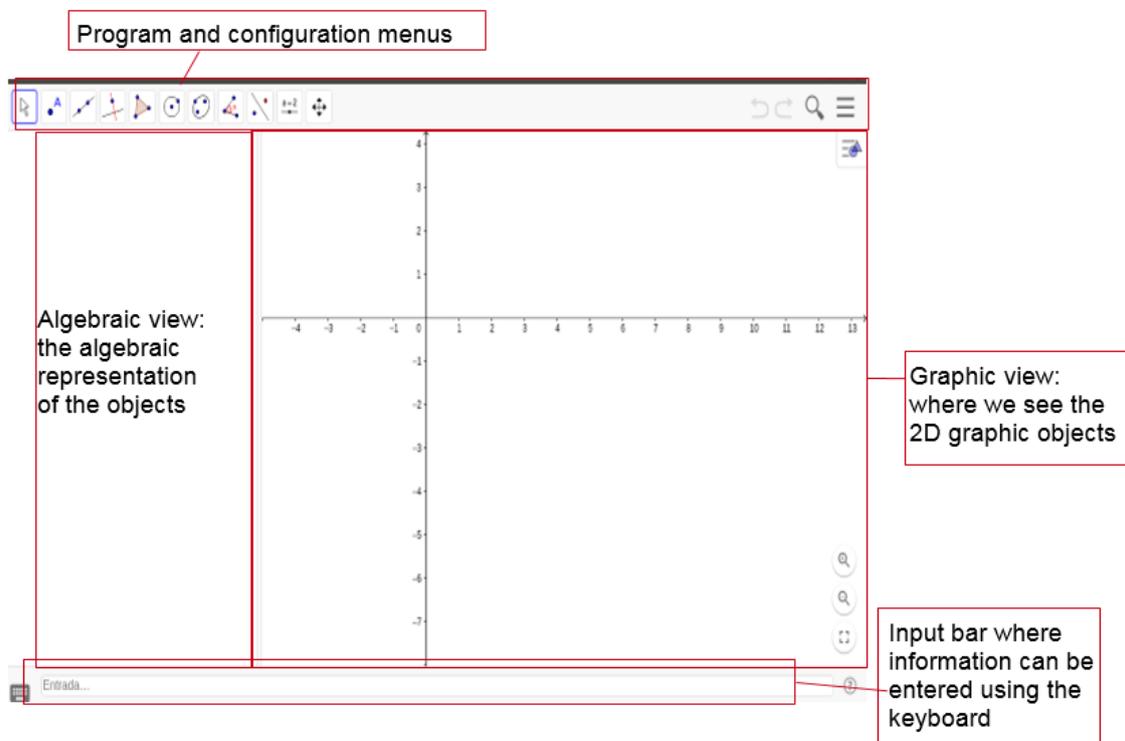
### Introduction

In this workshop we are going to use GeoGebra to represent some mathematical objects related with the visits to the *Geomining Museum*, *Natural Sciences Museum* and *the geometric walk*.

### WHAT IS GEOGEBRA?

GeoGebra is a dynamic geometry program that allows us to represent various objects and modify them once drawn, apart from other functionalities. We can work with the on-line version at <https://www.geogebra.org/apps/> or with the offline version if we have it installed on the computer or mobile phone (we can download it from the web <https://www.geogebra.org/download>).

When we open the program the interface shown is (on GeoGebra 6):



We will know that we have selected a certain function when its button appears boxed in dark blue:



The "**move**" function is selected.



The "**move**" function is not selected.

In this school the default language in the program is Spanish. We begin by changing the language to English in the configuration menu in the upper right corner  : **Configuration**→ **Global** → **Language**.

The input bar does not appear by default. When we need it we will have to make it visible. To do so, in the menu  we will select: **View**→ **Input bar**.

## Activity 1: PLATONIC SOLIDS

The five regular polyhedra are called platonic solids:

- Tetrahedron: has four triangular faces
- Hexahedron or cube: has six square faces
- Octahedron: has eight triangular faces
- Dodecahedron: has twelve pentagonal faces
- Icosahedron: has twenty triangular faces

GeoGebra has a command to construct these figures from two of its vertices (when we use the app, when we use the program we will need the vertices, or two vertices and a direction) so we only need to open the 3D view on the computer or the 3D Calculator app in the mobile.

In this case we are going to use mobiles or tablets in order to use the augmented reality (AR) application.

We propose you build each one of them using points  $(-1,1)$  and  $(-1,-1)$  as vertices, although you can choose other points:

Tetrahedron( $(-1,1),(-1,-1)$ )

Cube( $(-1,1),(-1,-1)$ )

Octahedron( $(-1,1),(-1,-1)$ )

Dodecahedron( $(-1,1),(-1,-1)$ )

Icosahedron( $(-1,1),(-1,-1)$ )

With the AR button try to situate the polyhedron on the real figure.

## Activity 2: LOGARITHMIC SPIRAL

The main characteristic of the logarithmic spiral is that the radius at each point (segment that connects it with the centre of the spiral) and the tangent line at that point form a constant angle. An approximation of it appears in certain natural phenomena such as the shapes of the galaxies, the arms of cyclones or the shell growth of some molluscs, such as Nautilus.

Possibly you have taken a picture of a Nautilus in the *Geomining Museum*. We will use it in the following construction (if you do not have a good photo, look for one on the internet).

We start by creating three **sliders**: **a**, **b** and **t**. The first two are the parameters of the curve and the third one will allow us to draw it.

To do this, just type the name of the parameter in the input window that appears in the algebraic view and press enter. By default it gives them values between

–5 and 5, but we will change them to the following values:

**a** between 0 and 10                      increment 0.01

**b** between 0.05 and 0.99              increment 0.01

**t** between 0 and 60                      increment 1

just double click on the –5 and we can change all the values.

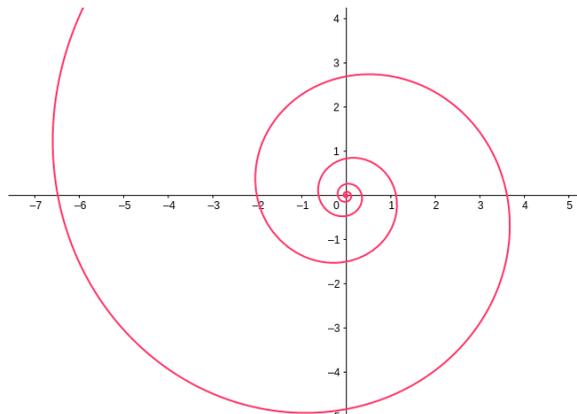
After this we create the curve using the input bar. We write "**curve**" and GeoGebra will suggest various possibilities. We choose:

**Curve**( <Expression>, <Expression>, <Parameter Variable>, <Start Value>, <End Value> )

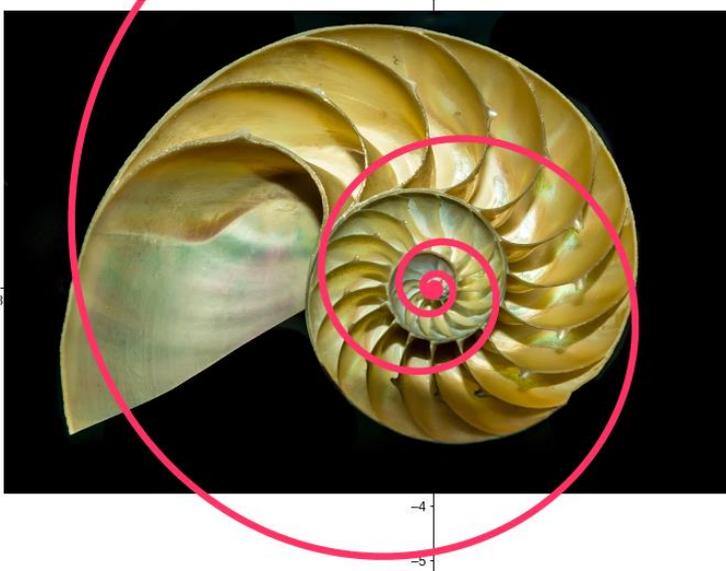
and we substitute the values with the parametric expression of the curve as follows:

$$\text{Curve}( a \cdot b^z \cdot \cos(z) , a \cdot b^z \cdot \sin(z) , z , -t , t )$$

We get a curve similar to the following one (when we move the parameters we see that its shape changes):



Now we will insert the image. In this upper menu we select "Image". When we click on the graphic view a dialogue box opens, and we can upload the image from the computer. The image will appear centred on the point where we clicked. We must place it as centred on the origin of the spiral we have drawn as possible



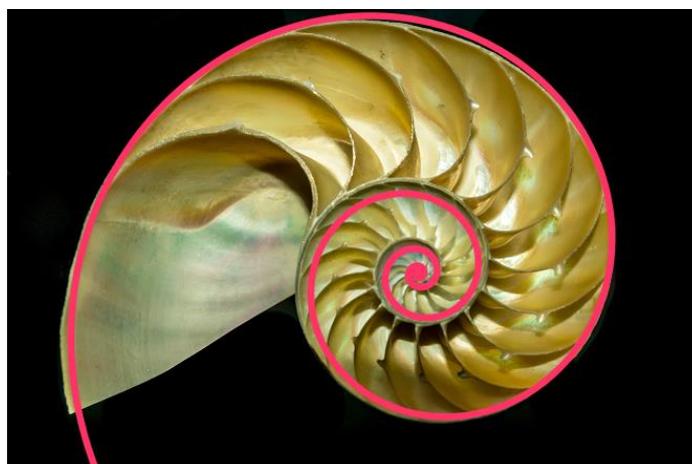
Depending on how our photo is, the spiral may be "upside down" compared to our photo. To change this we double click on the curve expression and add  $-\pi$  in the second coordinate:

$$b = \text{Curve}(a k^z \cos(z), a k^z \sin(z - \pi), z, -t, t)$$

$$\rightarrow \left. \begin{array}{l} x = 3.61 \cdot 0.83^z \cos(z) \\ y = 3.61 \cdot 0.83^z \sin(z - \pi) \end{array} \right\} -132 \leq z \leq 132$$

Finally, we change the colour and thickness of the spiral so that it looks correct. To do this we click the right button on it and go to **Settings**. In "**Colour**" we change the colour and in "**Style**" we change the thickness and type of line.

Now, moving the values of the three parameters we can adjust the curve quite well to the shape of the shell. We must take into account that it cannot always be adjusted perfectly, because the nautilus may have suffered blows, diseases, etc., which mean the growth of its shell has suffered irregularities.



### Activity 3: OVOID SHAPES

You have probably seen some dinosaur egg fossils or fossils of bird or reptile eggs in the *Geomining Museum* or in the *Museum of Natural Sciences*. Geometrically we can interpret an egg as a surface of revolution, that is, a surface that is generated by turning a curve around an axis.

We start by drawing an ovoid curve. There are different ways of drawing it, but we will use the following one:

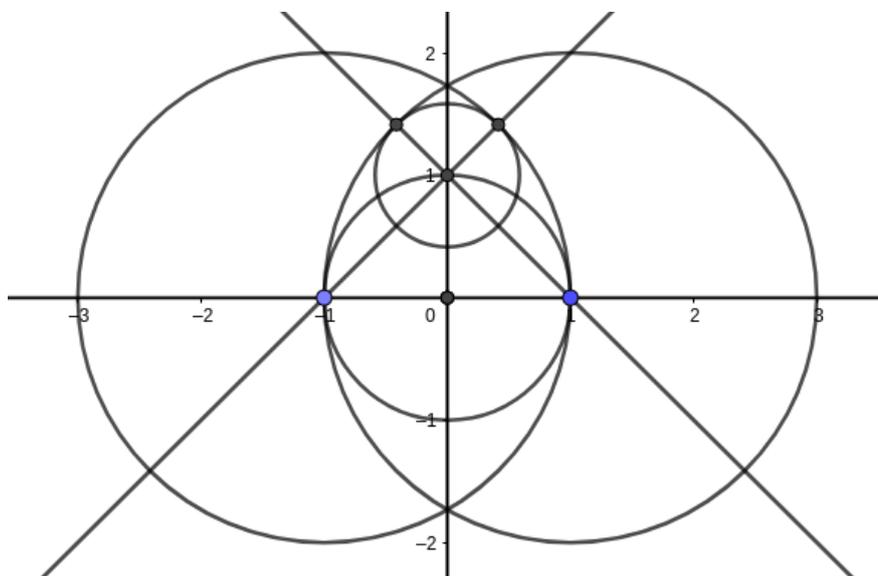
Let us mark the points  $(0,0)$ ,  $(1,0)$  and  $(-1,0)$ . Using the "Circle with centre through point" command  we draw the **circumferences**:

Centre  $(0,0)$  and passing through the point  $(1,0)$ ,

Centre  $(1,0)$  and passing through the point  $(-1,0)$ ,

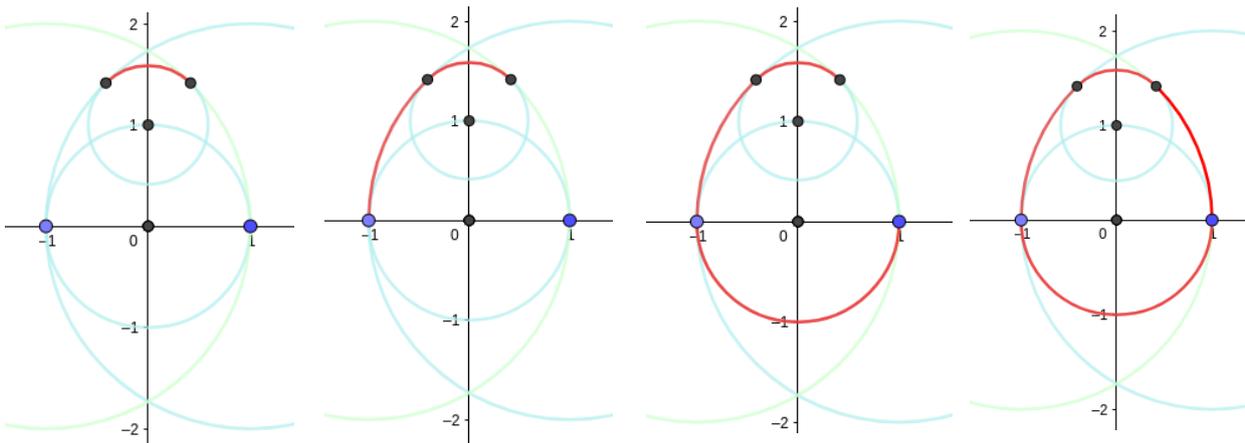
Centre  $(-1,0)$  and passing through the point  $(1,0)$ .

Let us mark the intersection of the first circumference with the vertical axis, that is, the point  $(0,1)$  and let us create the line passing through  $(-1,0)$  and  $(0,1)$  and the line passing through  $(1,0)$  and  $(0,1)$  with the command  **'Line'**. We mark the **intersections** of these lines with the biggest circumferences and finally we create the **circumference** with the centre  $(0,1)$  passing through these two points:



We will now draw the curve piece by piece. To do this, we begin by hiding the lines and changing the colour of the circumferences to a light colour. To do this we click with the right button on each of them and in **Settings**→ **Colour** we can choose another colour. This step is only to distinguish them from the arcs that we are going to draw next.

With the "Circular Arc" command we draw the arcs of the ovoid:



Centre at (0,1)

Centre at (1,0)

Centre at (0,0)

Centre at (-1,0)

We hide all the points and the circumferences to leave only the ovoid visible.

We now open the 3D graphics view. We find it in the menu in the upper right corner: **View→ 3D Graphics**. We see the ovoid curve in the horizontal plane.

Now we rotate the curve around the OY axis.

As we have drawn the curve into four pieces, we will have to rotate them separately. Notice that two of the arcs are symmetrical, so it will be enough to rotate only one of them, that is, it will be enough to make three pieces to have an egg.

In the input bar we write "**Surface**" and choose the option:

**Surface**( <Curve>, <Angle>, <Line> )

In the three cases the angle is  $360^\circ$  and the axis the yAxis:

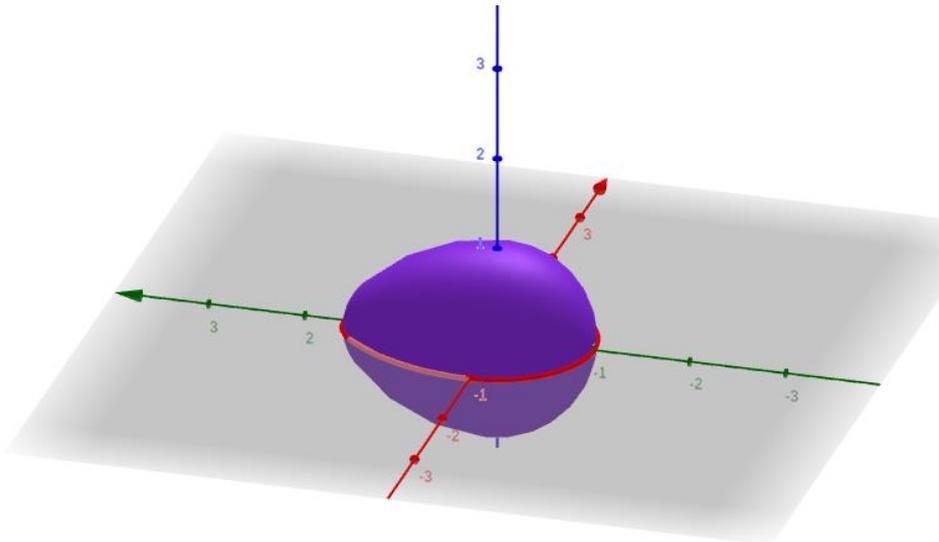
Surface(p,  $360^\circ$ , yAxis)

Surface(q,  $360^\circ$ , yAxis)

Surface(r,  $360^\circ$ , yAxis)

(The names GeoGebra has given to each arc may be different from these. Use the ones you have in your file).

Each part will appear with different colour and texture. To standardize them, go to **Settings** and give them the same colour and in **Style** give the option "**Line Thickness**" value 0. You will get a result similar to the following:



You can change the shape or size of the "egg" by moving the original points in the 2D graphic view.

#### Activity 4: **BASALT COLUMNS**

Many minerals in their solid form crystallize in a geometric structure, such as the pyrite, which crystallises in cubes, the topaz that takes the shape of square prisms or basalt, which forms hexagonal prisms.

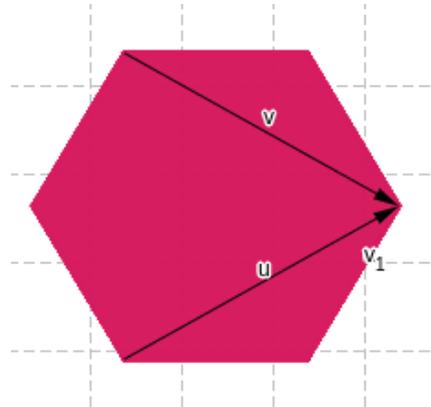


Giant's Causeway in County Antrim on the coast of Northern Ireland

We will represent a structure of basaltic columns. To do this we will create a column and then we will translate it to create a structure.

We must open the 2D and the 3D graphic views.

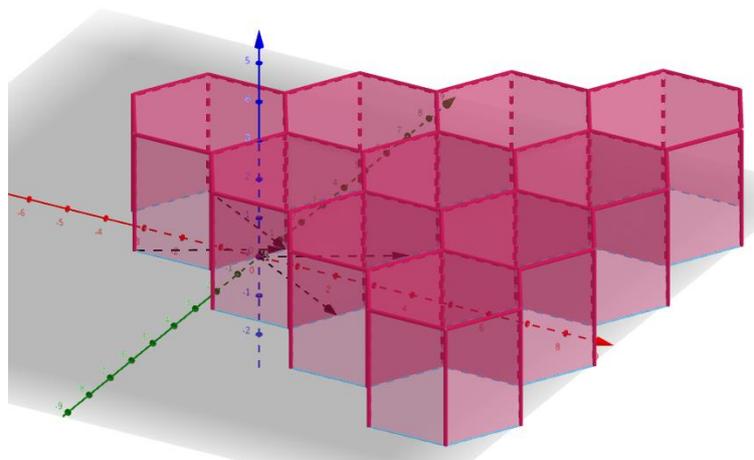
In the 2D graphic view we create a regular hexagon with the button "**Regular polygon**"  and then the vectors  $u$  and  $v$  with :



Let us create a **slider** " $g$ " with  and give it values between 0 and 1 with increment 0.1.

Now we go to the 3D graphic view, where we create a prism of height  $3g$  with the button . When we move the **slider** we see that the prism grows on the hexagon.

Leave the slider with value  $g=1$  for a moment and with the tool "Translate by vector" , we translate the prism first using  $u$ , and then again using  $v$ . After this, we translate the new prisms again with both vectors, and repeat the process as many times as we want, until we obtain something like this:



Now, by setting the slider to 0 and moving it, we will see how the columns grow and get in their place.